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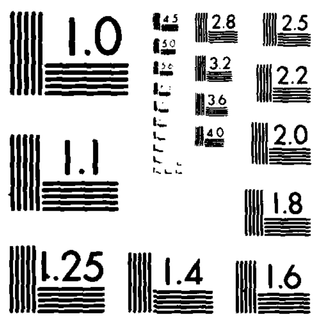

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CONDITIONS FOR TRANSITIVITY OF MAJORITY RULE WITH ALGORITHMIC INTERPRETATIONS

by

Herve Raynaud

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CONDITIONS FOR TRANSITIVITY OF MAJORITY RULE WITH ALGORITHMIC INTERPRETATIONS*

by

Herve Raynaud**

1. Introduction

Using particular cases of the condition Ref (v,i,j) (in Kolher [1978]), one can derive nine conditions for transitivity of the method of majority decision which all possess a generating algorithm with possible sociological or economical interpretations. Some celebrated conditions appear to be particular cases of this structure. Some other, "new conditions," at least new under their algorithmic form, suggest explanation for some efficient committee votings.

2. Preliminaries

In all what follows:

- X denotes a finite set of objects (or alternatives);
- E (or $E(X)$) denotes a profile on X, i.e. a sequence $\theta_1,..\theta_N$ of N total orders on X called individual orders;
- $\forall Y \subseteq X$, $\theta_i(Y)$ is the restriction of θ_i to only the objects in Y and $E(Y)$ is the restriction of the profile E to the objects of Y, i.e. the sequence of the $\theta_i(Y)$'s;

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- A CTMM is a Condition for Transitivity of the Method of Majority decision;
- All the CTMM in this paper will consist in the prohibition of certain profiles.

If R is a given reference order, and i, j, ν, three integers such that $0 < i \leq \nu + 1$, $0 < j \leq \nu + 1$, then Kölher's [1978] condition Ref (i,j,ν) says: "$\forall Y \subseteq X$, $|Y| = \nu + 1$, the i-th object in $R(Y)$ is never in the j-th rank in any $\theta(Y)$.

We shall specify here the value of $\nu = 2$. Nine conditions Ref can be then written, i and j being able to take the values 1, 2, 3. These conditions will be denoted C_{ij} and, R being fixed, depend only on i and j.

The reader familiar with voting theory will not be surprised to know that a previous title of this paper has been "oriented value restriction conditions." The reason of the rejection of this title is simple: it did not point out the most interesting fact which stands in the given algebraic and socioeconomic interpretations. The so-called value restriction conditions (cf. for instance Sen [1966]) do not take into account any specific reference order. They say: "For each triple T, one of the three objects is never in one certain rank in $E(T)$."

We are here taking a particular case of these conditions as, being given a reference order R, there is an i and a j common to all triples such that for any of them, say T, the i-th object in $R(T)$ will never be j-th in $E(T)$.

We have described in another paper algorithms for generating maximal profiles following the value restriction condition (Raynaud

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[1980]), but these algorithms do not offer a clear interpretation in economic or sociologic terms.

On the contrary, as i and j can take values 1, 2, 3, nine conditions Ref with interpretations (for at least eight), can be written. These conditions will be denoted C_{ij}, and R being fixed, they depend only from i and j.

The reader familar with voting theory will not be surprised to know that the C_{ij}'s are particular cases of the celebrated Ward's CTMM.

Ward's Condition: A profile is said to follow Ward's condition if and if one cannot find three objects a, b, c in X and three individuals orders θ_1, θ_2, θ_3 such that

$$\theta_1(a,b,c) = a,\ b,\ c$$
$$\theta_2(a,b,c) = b,\ c,\ a$$
$$\theta_3(a,b,c) = c,\ a,\ b$$

This condition is often described saying: there is no Condorcet triple. This ensures transitivity of majority voting for any odd number of individual orders.

Proof: Let E be a profile following Ward's Condition. If strictly more than 50% of the individual orders rank a before b, and strictly more than 50% b before c, it is not possible that strictly more than 50% of the individual voters rank c before a.

If it was so, on the contrary, one would have one vote at least ranking a before b before c, another one ranking b before c

before a and a third one ranking c before a before b and E would not follow Ward's Condition.

Proposition 1: C_{ij} is a particular case of Ward's Condition.

Proof: For every triple $T(a,b,c)$ two different permutant latin squares can be found in $E(T)$:

$$\begin{matrix} a & b & c \\ b & c & a \\ c & a & b \end{matrix} \quad \text{and} \quad \begin{matrix} b & a & c \\ a & c & b \\ c & b & a \end{matrix}$$

Any of the C_{ij} forbids one vote at least in each of the two possible latin squares (trivially checked by mere enumeration).

In order to describe the correspondences between the properties of the C_{ij}'s, we need to recall some tools.

Definition 1 (Romero, [1978]): Let C be a CTMM. I say that E follows $\overline{C}$ or C in-the-mirror if the set of orders opposite to the orders of E follows C. (This is identical to what Peter C. Fishburn [1972] calls dual or converse orders and profiles.)

It is trivial to remark that $\overline{C}$ is a CTMM. If $\overline{\theta}$ denotes the order θ in-the-mirror, $\overline{E}$ denotes the profile $(\overline{\theta}_1, \overline{\theta}_2, \ldots, \overline{\theta}_N)$. In addition N_C, maximum number of different votes in a profile following condition C is equal to $N_{\overline{C}}$.

Definition 2: Let R be a reference order on X and θ a total order on X. With any pair (θ, R), one can associate a matrix $M_R(\theta)$ with n^2 elements denoted P_{ij}, $P_{ij} = 1 \Longleftrightarrow$ the i-th object in R is the j-th object in θ. $P_{ij} = 0$ in all other cases.

If $M_R^t(\theta)$ denotes the transpose of $M_R(\theta)$, $M_R^t(\theta)$ is equally a permutation matrix and can be written $M_R(\theta^t)$ in a unique way. θ^t is the transposed order of θ; $E^t = (\theta_1^t,\ldots,\theta_N^t)$ is the transposed profile of $E = (\theta_1,\ldots,\theta_N)$.

The transposition operation has been used equally by various authors under different names: inversion, duality, etc.... For more details on Definitions 2 and 3 and Proposition 2, the reading of Köhler [1978] will be enlightening.

<u>Definition 3</u>: E follows the CTMM C^t if and only if E^t follows C.

<u>Proposition 2</u>: $N_C^t = N_C$.

<u>Proof</u>: Trivial, from the fact that transposition is a bijection of the set of total orders into itself.

<u>Proposition 3</u>: $C_{ij} = C_{ij}^t$ $(= (C_{ij}^t)^t)$ $C_{ij} = \overline{C_{ik(j)}}$ with $k(j) = 4-j$

<u>Proof</u>:

(1) Let us consider θ in a profile E following C_{ji}. Consider the 3×3 submatrix of $M_R(\theta)$ corresponding to the objects (x,y,z) of a triple T. If, for T, the j-th object in $R(T)$ is not the i-th in θ, then the corresponding ji-th element in the submatrix is zero. If I transpose $M_R(\theta)$, the considered submatrix still corresponds to the objects of T, but the submatrix has been itself transposed, and it is its ij-th element which is a zero. As R has stayed the same, in θ^t one can say that the i-th object is never in the j-th rank.

(2) Consider E following C_{ij}. Let T be a triple of X, and x be its i-th object in $R(T)$. Let $\theta \in E$. It is clear that in $\bar{\theta}(T)$, x

- will never be in position 1, if it was never in position 3 in θ,
- will never be in position 2, if it was never in position 2 in θ,
- will never be in position 3, if it was never in position 1 in θ.

The result follows.

The nine conditions that we are going to describe will appear as clearly grouped by the operators (transposition and in-the-mirror) in four categories

1) C_{21} and C_{23} (C_{23} = C_{21} in-the-mirror)
2) C_{32} and C_{12} which are the transposed of the previous ones.
3) C_{11} and C_{13} (for C_{13} = C_{11} in-the-mirror)
   C_{31} (transposed of C_{13}) and C_{33} (= C_{31} in-the-mirror)
4) C_{22}: auto in-the-mirror and auto-transposed.

The nine conditions have been studied from three different points of view:

1) What are their algorithmic interpretations?
2) Given a set of orders, is there a good algorithm allowing the determination of R such that the given orders follow C_{ij} with regards to R?
3) What is the maximum number of different votes in a profile following the condition?

3. C_{21} and C_{23}

(1) From the very well known algorithm for building all the possible individual orders in a profile satisfying C_{23} (Arrow's and Black's [1963] condition--take the last objects among the extremes of $R(X)$, then the penultimate ones among the extremes of R (the remaining objects), etc... --one deduces the following for C_{21}: take the first objects among the extremes of $R(X)$, then the second ones among the extremes of R (the remaining objects), etc....

The proof of the algorithm is straightforward: the object chosen as first object is the first in all the triples containing it. Hence, it cannot be second in any triple which contains it and has to be chosen among the extremes of $R(X)$.

The same property is true for the p-th ranked object with regards to $X - \{$the $(p-1)$ already chosen objects$\}$.

(2) This algorithm, of course, has many easy interpretations. One translates the algorithm in a kind of dual form:

1) Choose the worst object. Rank it last.

2) Amongst the neighbors of this object in R, choose the penultimate. Etc....

Compared to the usual description of the Blackian ordering, this ranking is done in function of disgust instead of appeal, but follows the same logic.

(3) A relevant algorithm to find a reference order R, given a sequence $(\theta_1,\ldots,\theta_N) = E$ of total orders such that E follows C_{23} with regards to R has been given by Romero [1978] under the name of pyramid algorithm. Description and proof are too long to be repeated here. Reversing the orders, it applies clearly to C_{21}.

(4) Additionally, the fact that $N_{C_{21}} = N_{C_{23}} = 2^{|X|-1}$ is classical and comes from 3(1).

4. C_{32} and C_{12} Offer More New Results

Let us recall explicitly their definition.

C_{32} (resp. C_{12}) $\equiv$ for all triples T of objects of X, the last (resp. first) object in $R(T)$ is never second in any $\theta_i(T)$.

(1) An algorithm which is clearly a particular case of Kölher's algorithm "tri" can be easily derived from this definition: in order to build all the possible different orders in a profile E following C_{32} (resp. C_{12}), one should not rank the last (resp. first) object in R in the second rank of any triple in θ_i; hence, this object will be ranked in one of the two extreme position, at rank 1 or n.

For the same reason, the penultimate (resp. second) object of R can only be ranked at the first or the last one of the ranks remaining free, etc....

(2) One sees that a dual interpretation can be easily given here too. Consider C_{12}: each individual agrees on the reference order which can be, for instance, a ranking according to efficiency in terms of audience retained by TV programs. Then, each individual is able to divide the set of programs in two classes, one being the class of programs which the individual judges as likely to have a good influence on listeners, and the other class in the contrary is judged as likely to have a bad influence. Then, the issue which among the "goods" has the largest audience will be ranked the best, whereas the "bads" will have as bad a ranking as a large audience.

In other--and more formalized--words, for each individual vote θ_i there exists a partition of X into two classes X_1, X_2; the first class contains the objects ranked in the same order in θ_i and R, when the second contains the objects ranked in θ_i according to $\overline{R}$, the last object of the first class being ranked in θ_i before the objects of the second class.

It is clear that C_{32} which is equal to $\overline{C_{12}}$ is subject to a similar interpretation--"good" being changed to "bad" and "worst" to "best" at the appropriate places.

(3) A relevant algorithm to describe all the possible reference orders (for which a sequence $\theta_1 \ldots \theta_N$ of total orders on X can be considered as a profile following C_{32}) is easily derived from 4(2).

Let a, b, ..., z be an individual order θ_i. The last object in the reference order has to be a or z. Hence, two possibilities at most can be kept for the last object in the reference order, and this result can be the root of an oriented binary tree, the paths of length $(|X| - 1)$ in it being the solution possible reference orders. One checks, then, on the other votes whether a and z both still can be last in R. Then, in order to determine the penultimate, one does the same with the remaining objects, once the last one is determined, etc.... At each step, three events can occur: two objects are extremals, one or zero--this last case being the signal of impossibility.

Example: Problem: Determine all reference orders R such that E follows C_{32}

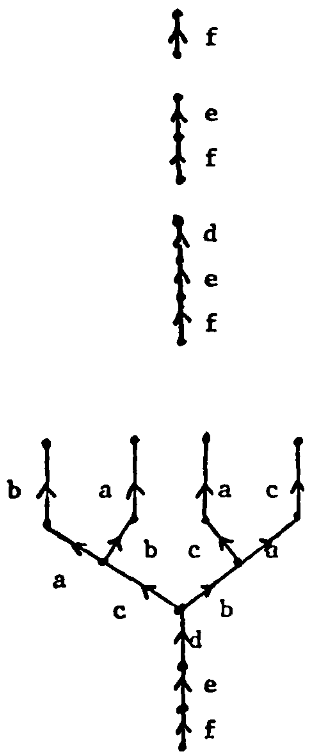

Fig. 1

1) E(abcdef) = ((fdcabe), (cabdef), (febacd))

Only f is extremal in all the votes, and can, hence, be last in R

2) E(abcde) = ((dcabe), (cabde), (ebacd))

Only e is extremal

3) E(abcd) = ((dcab), (cabd), (bacd))

Only d is extremal

4) E(abc) = ((cab), (cab), (bac))

b and c are both extremals.

5) All the remaining pairs of elements can be ranked arbitrarily in the two first ranks.

The problem has 4 solutions for R

- b a c d e f
- a b c d e f
- a c b d e f
- c a b d e f

The reader who already knows the pyramid algorithm will maybe think that, this one being easier, why not transpose first, check if E^t follows C_{32} and transpose again. It is not so simple, because the only thing that can be said is the following:

<u>Proposition</u>: If tR means that the operation "transpose" on orders has been done with reference order R then: E follows C_{23} with regard to $R \Longleftrightarrow E^{tR}$ follows C_{32} with regard to R.

Proof: Straightforward: Each 3×3 submatrix of $M_R(\theta)$, done from rows i, j, k and the columns where the "1's" stand, has a zero in the second element of the third column.

By transposition, the second element of the third row becomes nul, which means that E^{tR} follows C_{32}.

(4) The generative algorithms give straightforwardly

$N_{C_{32}} = N_{C_{23}} = 2^{|X|-1}$

4. C_{31} and C_{11} (= C_{31} with reverse reference order)

$C_{13} = C^t_{31}$ and C_{33} (= C_{13} with reverse reference order = C_{31} in-the-mirror). These conditions have very similar algorithm patterns. For example, the fact of taking a reverse reference order has no significant meaning from the point of view of applications where you can arbitrarily rank objects as well from the first to the last or from the last to the first.

(1) Let us consider C_{13}. The first object in R is first of all the triples which contain it. As it should never be third, it can only be first or second in an individual order. Once it is ranked, the same reasoning can be repeated for the second object in R and the remaining free rankings. The algorithm comes from this directly: give the i-th object in R the first or the second of the remaining free ranks, then go to the (i+1-th) object in R.

How can this be interpreted? One can of course think of voting for any definitive ordering when one supposes it will not be far from some certain reference order R that one already possesses.

This will occur, for instance, in administration, where candidates to a repartition through the country will be judged by a committee, being given the fact that experience in the job is a dominant criterium, and indicates a reference order R, which is going to be slightly modified taking efficiency in account as a secondary criterium.

We have thought, on the other hand, of a very special technical situation which could be described within the frame of C_{31}.

N Workers performing a big job divided into n tasks are going to take the decision of the order according to which the tasks will be achieved. All the tasks have to be done. The supplies for task i reach the yard on the i-th day: because the first day is used to discuss procedure and other administrative work, on the beginning of the second day, they have the choice of performing task one or task two; on the beginning of the third day they will have the choice between the one of the first two they have not done the day before and the third one, and so on.

The interpretations for the two other cases can be described in terms of appeal versus repel and conversely.

(2) A dual description can be done too. Consider C_{13}, for example. The last object in a vote cannot be the first in R, in any of the triples in which it is included. For this, it has to be chosen from the two last objects in R. In the same way the penultimate will be chosen from the two last remaining objects in R, etc.... Conversely C_{31} can be described saying

- take the last object in R and rank it in one of the two last ranks,
- take the remaining last one in R and rank it in one of the two remaining last ranks, etc....

(3) Because of these symmetry reasons, only the algorithm for finding the fitting R's for C_{13} will be described:

Algorithm

(1) is there an object always in the two first positions? If no, the problem has no solution. If yes, R begins by one of the possibly two objects always in rank 1 or 2.

(2) is there an object always in the two remaining positions? If no, the problem has no solution. If yes, R proceeds by one of the possibly two objects always in the two remaining ranks 1 or 2, etc....

Example: E = (bcdae, acbed)

- 1st object is c; remaining votes are b d a e, a b e d
- 2nd object is b; remaining votes are d a e, a e d
- 3rd object is a; possible reference orders are c b a e d and c b a d e.

(4) Clearly, this time again $C_{31} = C_{13} = C_{11} = C_{33} = 2^{|X|-1}$

5. Then Remains the Original C_{22}

Why is it so original? You would think that, given a reference order R, $2^{|X|-1}$ different individual orders at a maximum can follow

C_{22} with respect to R? Let us check it for $X = \{a,b,c\}$. It works: if (a,b,c) is the reference order, $\{bac, bca, cab, acb\} = E^3_{Max}$.

If you now try to obtain by enumeration the maximum number of votes satisfying C_{22} for four objects, you can do it introducing in the previous orders of E^3_{Max} a fourth object, let us say d, then deleting the orders for which a triple makes an impossibility. One obtains this way:

~~d b a c~~	b d a c	b a d c	~~b a c d~~
~~d b c a~~	~~b d c a~~	~~b c d a~~	~~b c a d~~
~~d c a b~~	c d a b	c a d b	~~c a b d~~
~~d a c b~~	~~a d c b~~	~~a c d b~~	~~a c b d~~

Legend:

deleted because of a b d

deleted because of a c d

deleted because of b c d

kept

And if, now, you try to introduce a 5th object e, you obtain

~~e b d a c~~	~~b e d a c~~	~~b d e a c~~	~~b d a e c~~	~~b d a c e~~
~~e c d a b~~	~~c e d a b~~	~~c d e a b~~	~~c d a e b~~	~~c d a b e~~

Legend:

deleted because of e d a

deleted because of b d e

deleted because of c d e

In other words, there is <u>no</u> profile on X, with $|X| \geq 5$ which satisfies C_{22}. The conjecture is: this has a nice economical interpretation!

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